US Nobel Laureates: Logistic Growth versus Volterra-Lotka

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Abstract

The logistic-growth equation is a special case of the Volterra-Lotka equations. The former describes competition only between members of the same species whereas the latter describes competition also with other species. In the study of US Nobel laureates considering laureates *per population* improves the quality of the logistic fit but the Volterra-Lotka approach suggests that a logistic description would be a good approximation for data *per unit of time* rather than *cumulative* data. Fitting logistic S-curves on cumulative data — although proven successful in many business and other applications — constitutes treacherous terrain for inexperienced S-curve enthusiasts. The Volterra-Lotka analysis of Nobel laureates reveals other insights such as that Americans and other nationalities are locked in a win-win struggle with Americans drawing more of a benefit, and also that American Nobel laureates "incubate" new Nobel laureates to a lesser extent than other nationalities.

Keywords: logistic growth, S-curve, Volterra-Lotka, US Nobel laureates, World Nobel laureates, competition, win-win

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1. Introduction

It has been suggested that the competition for Nobel Prize awards can be described by logistic-growth curves.[1] My first attempt fitting a logistic to the cumulative number of US Nobel laureates in 1988 concluded that the US Nobel niche was already more than half full and implied a diminishing annual number of Nobel Prizes for Americans from then onward.[2] Ten years later I confronted those forecasts with more recent data in my book *Predictions – 10 Years Later*.[3] The agreement was not very good. The forecasts fell below the actual data and despite the fact that there was agreement within the uncertainties expected for a 90% confidence level the discrepancy did not go unnoticed. A technical note published in this journal in 2004 highlighted the inaccuracy of my forecasts and cast doubt in the use of logistics to forecast US Nobel laureates.[4] On my part, I refit the updated data sample with a new logistic pointing to a higher ceiling and began wondering whether there was evidence here for the known bias of logistics to underestimate the final niche size. The new forecast again indicated an imminent decline in the annual number of American Nobel laureates.

Years later while preparing a new edition for my book — *Predictions – 20 Years Later* — I once again confronted forecasts with data. The situation turned out to be the same as ten years earlier, namely the forecasts again underestimated reality and despite agreement with the result of ten years earlier within the uncertainties expected for a 90% confidence level there was now clear disagreement between recent actual numbers and the original forecasts of twenty years earlier. The situation was reminiscent of the celebrated Michele-parameter episode in experimental physics where a measurement repeated many times over the period of fifty years kept reporting an ever-increasing value always compatible with the previous measurement but finally ending up in violent disagreement with the very first measurement.

So in this paper I want to settle the question of the ever-growing ceiling of the logistic curve fitted to the US Nobel laureates once and for all.

2. Historical/Theoretical Considerations

Logistic growth — yielding S-curves — originally conceived to describe the growth of species populations in nature has also been extensively employed throughout the 20th century to describe and forecast animate and inanimate populations stemming from social activity. Alfred Lotka in the early 20th century, Cesare Marchetti in late 20th century and many other scholars have quantitatively applied the principle of Darwinian competition — survival of the fittest — via its logistic-equation formulation to obtain descriptions and forecasts for the widest range of growth processes. Despite the fact that the mathematical formulation is derived from species competing in nature, the analogue to social phenomena and competition among inanimate populations has been demonstrated to be valid.[5] Forecasts made in this way enjoy scientific objectivity, i.e. they are free of the human bias that typically plagues forecasts based on socio-politico-economic theories most of which are founded on beliefs and opinions of experts.

In this paper we address the process of winning Nobel Prizes. It constitutes a competitive process because Nobel Prizes are desirable and at the same time they are a "limited resource" with a restrained number of them being awarded each year. By definition, the best-fit candidates win. Obviously, a peace Nobel Prize is very different from a Nobel Prize in Physics. Moreover, some prizes may be shared among as many as three individuals whereas others are given to only one individual. Following Marchetti's first attempt to forecast US Nobel laureates here too each laureate is counted as one independently of what discipline he or she was in and independently of how many colleagues shared the prize. The justification for this is that we are counting individuals with exceptional contributions to the benefit of mankind and on the average relative underachievements are compensated for by relative overachievements.

In the Volterra-Lotka system of equations logistic growth for two or more species has been generalized with cross terms and coupling constants. This formulation describes not only competition among the members of the same species but also how the species' rate of growth will be affected by the presence of another species. For our Nobel-Prize study the first-order approximation of a 2-species world has Americans competing against all others.

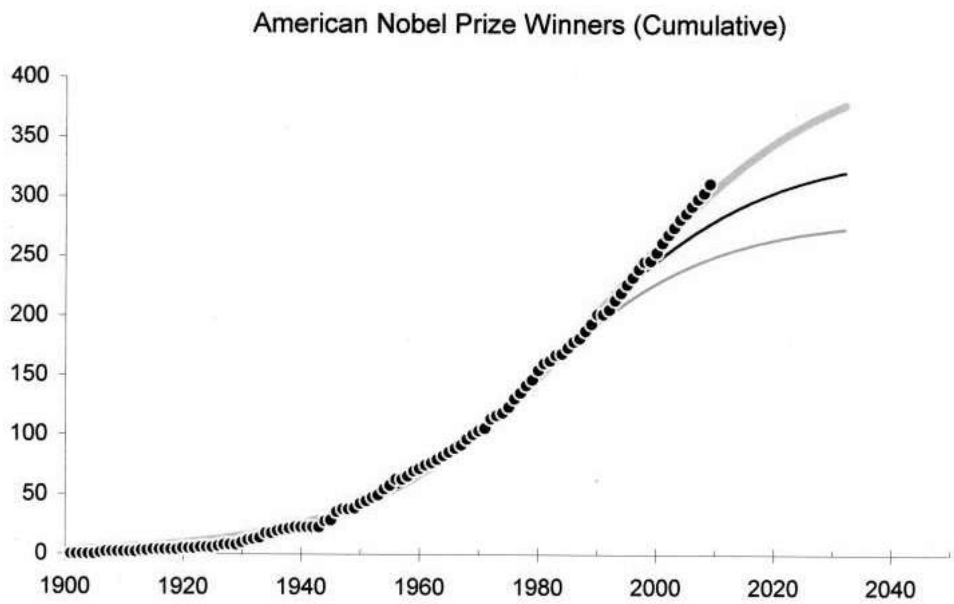
Generalizing the concept of competition for Nobel Prizes to national competition is not justified only a posteriori from the goodness of the model description. National competition emerges spontaneously as it does in the Olympic Games for non-team sports. Neither for Nobel Prizes nor for non-team sports is there national competition in the corresponding institution's philosophy. But in both cases there is national support for the contenders and national pride and rewards for the winners.

3. Logistic-Growth Fits

The data come from the Nobel Foundation. Individuals with double nationality were classified as nationals of the nation where the research for which they were being distinguished was accomplished. The cumulative data were fitted to the logistic S-shaped pattern given by Equation (1). The fitting procedure involved the minimization of a Chi-Square using EXCEL's solver. For the errors needed in the calculation of the Chi-Square purely statistical errors were assumed for each annual data point as elaborated in Reference [6].

$$X(t) = \frac{M}{1 + e^{-\alpha(t - t_o)}} \tag{1}$$

Figure 1 shows the three logistic fits respectively performed on data of the periods 1900-1987, 1900-1998, and 1900-2009. The quality of the fits as judged by the values of the reduced Chi-Square (i.e. Chi-Square per degree of freedom) becomes worse as the data set increases with time. The results are tabulated in Table I.



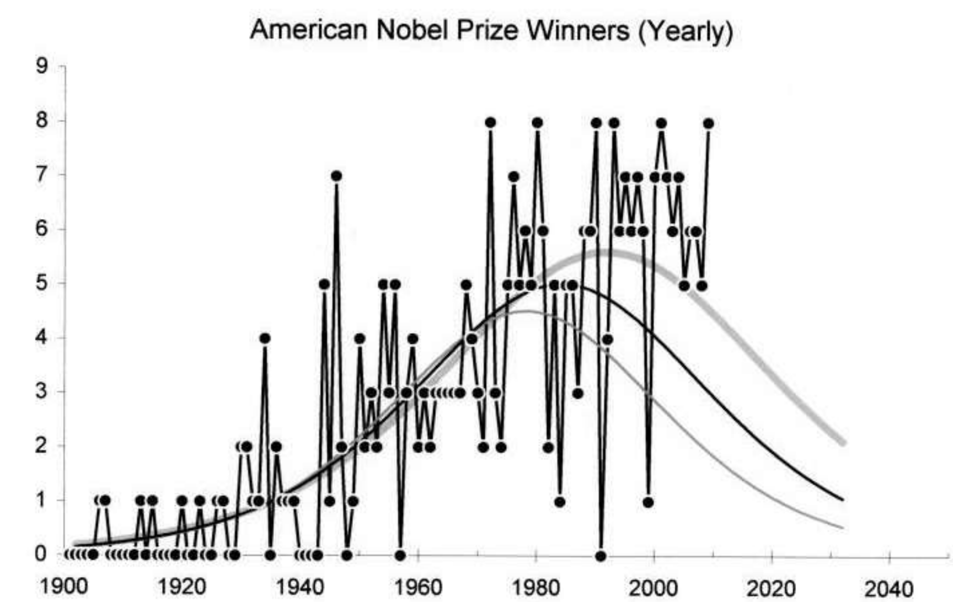


Figure 1. At the top cumulative data and logistic fit for three different periods: 1900-1987 (thin gray line), 1900-1998 (thin black line), and 1900-2009 (thick gray line). At the bottom, the life cycles (annual numbers) corresponding to the curves and the data at the top.

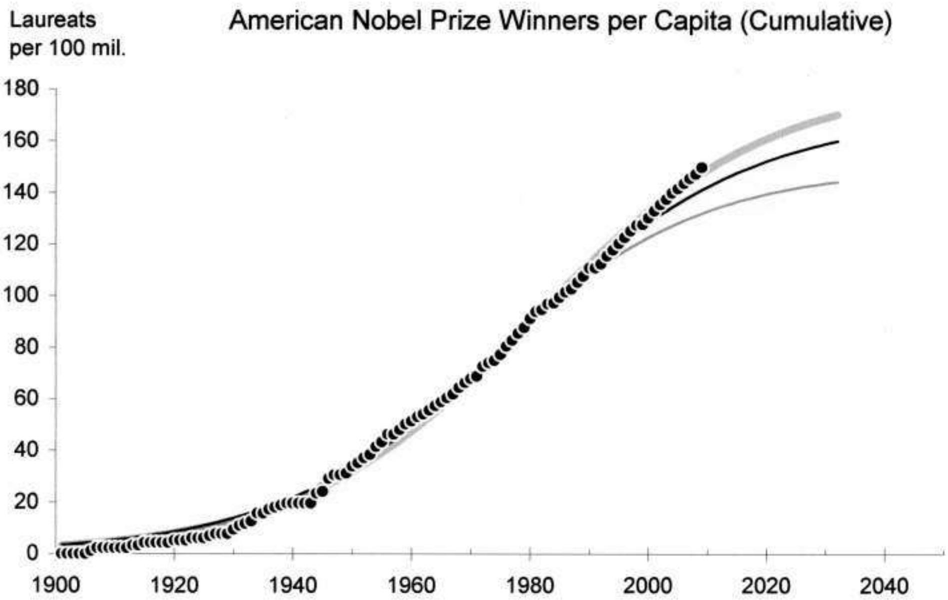
The ceilings of the logistic curves were found to be increasing with time in a significant way. We can estimate the uncertainties on the three values of M as $\pm$ 20% with 90% confidence from an extensive Monte Carlo study on uncertainties expected for logistic fits.[6] Within these uncertainties there is agreement between the M of successive time periods but disagreement between the first and the last period.

One explanation for M to be constantly increasing is the fact that the US population itself has also been increasing over the same historical period. An increasing population provides an increasing "niche" for Nobel Prize winners. In fact, Equation (1) is the solution of the logistic growth equation — Equation (2) below — that can be solved *only* if a and M are constants.

$$\frac{dX}{dt} = aX\left(M - X\right) \qquad (2)$$

Consequently a logistic S-shaped pattern for $X(t)$ is presumptuous and probably wrong if M increases with time. This could be a reason for the poorness of the fits.

An obvious way to account for the growing American population would be to study the number of laureates *per capita* thus rendering M time-independent and Equation (2) solvable. If we then repeat the previous analysis for Nobel Laureates normalized to population we obtain new results tabulated in Table II and plotted in Figure 2.



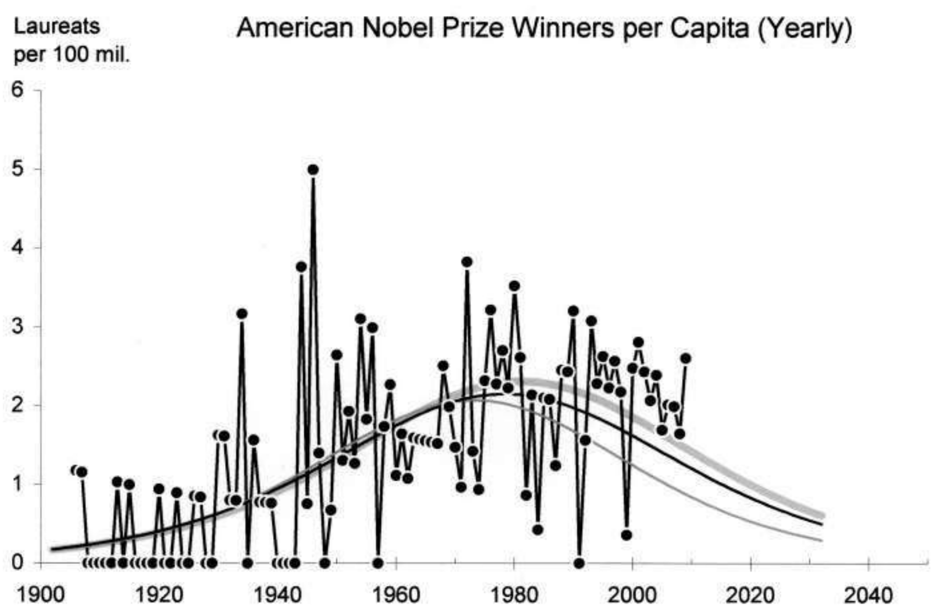


Figure 2. At the top cumulative data and logistic fit for three different periods: 1900-1987 (thin gray line), 1900-1998 (thin black line), and 1900-2009 (thick gray line). At the bottom, the life cycles (annual numbers) corresponding to the curves and the data at the top. The data points here are those of Figure 1 divided by the US population at the time.

The quality of the fits is better now and there is consistency, namely all values of M are within the expected uncertainties of ± 20% from each other. In the lower graph of Figure 2 there is even some evidence that the annual normalized number of US Nobel laureates has indeed begun decreasing.

Yet, there is still some tendency for *M* to increase with time and for confidence levels to decrease in longer data sets. Also we obtain counterintuitive forecasts for a dramatic decline of American Nobel laureates and/or a major increase of the American population by the second half of the 21st century. The tendency of M to still grow — as one of the reviewers of this article suggested — could be linked to the fact that current laureates build more directly on the work of recent laureates so that in a way the S-curve feeds on itself as it unfolds thus increasing its ceiling.

Table I - Results for Logistic Fits on US Nobel Laureates

Historical period	M	α	t_o	X^2	Degrees of freedom	Confidence level
1900-1987	332.08	0.057641	1983.139	55.37	76	96%
1900-1998	395.87	0.052544	1989.415	101.82	87	13%
1900-2009	490.64	0.047131	1997.851	147.47	98	<1%

Approximate confidence levels are calculated from the values of the reduced X^2. Only statistical errors considered.

Table II - Results for Logistic Fits on US Nobel Laureates per capita

Historical period	M	α	t_o	X^2	Degrees of freedom	Confidence level
1900-1987	151.76	0.054584	1973.2	41.25	76	>99%
1900-1998	172.10	0.049879	1978.276	55.63	87	99%
1900-2009	196.33	0.045044	1984.234	87.31	98	77%

Approximate confidence levels are calculated from the values of the reduced X^2. Only statistical errors considered.

4. Volterra Lotka

Let us go back and rethink the "physics" of the situation. Logistic growth describes the evolution of a population growing in competition, where the competition is between the members of the population, like rabbits competing for grass in a fenced-off range. In our case the species is US Nobel laureates. But besides the competition amongst them there is also competition between Americans and nationals of other countries. To the extent that US Nobel laureates represent about half or more of all Nobel Prizes every year, it is a good approximation to consider a duopoly i.e. a niche with only two species: Americans and all others grouped together. The species "all others" is rather inhomogeneous but with US Nobel laureates and all Nobel laureates both being well defined as species candidates, "all others" also becomes a well-defined species candidate.

Populations growing in competition in a two-species niche have been described by the Volterra-Lotka system of equations and are fairly straightforward to study.[7-10] The system consists of two logistic-growth equations like Equation (2) coupled with cross terms involving each other's population, see Equations (3) below. The new constants c_{xy} and c_{yx} determine how one's rate of growth depends on the presence of the other.

$$\frac{dX}{dt} = a_x X - b_x X^2 + c_{xy} XY$$
$$\frac{dY}{dt} = a_y Y - b_y Y^2 + c_{yx} XY \qquad (3)$$

Because there are large fluctuations on the yearly data the numbers were grouped together inside time bins of decades before the analysis. Equations (3) were evaluated numerically. The six constants were determined via a global fit to Equations (3) using EXCEL's solver to minimize a Chi Square again considering only statistical errors. In the fitting procedure two more constants were varied, the first-decade points; these starting values correspond to integration constants. Thus the total number of parameters varied was eight (the six parameters in Equations (3) plus the two starting values).

The fit is of acceptable quality and the results are graphed in Figure 3 and tabulated in Table III. The American trajectory is S-shaped (but not logistic) and the long-term forecast is roughly a 50-50 split of all Nobel Prizes between Americans and all other nationalities.

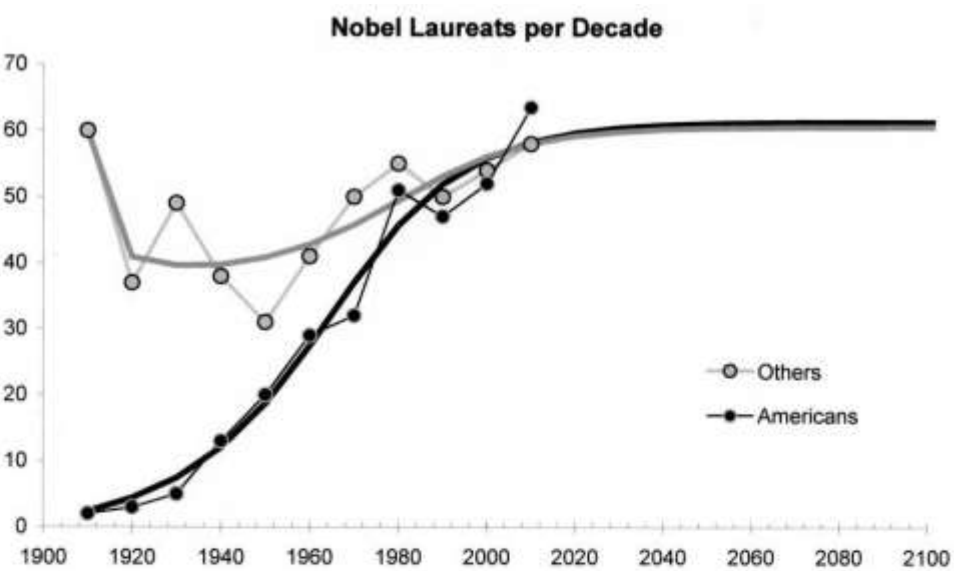


Figure 3. Decennial data points and solutions to the Volterra-Lotka equations (the last data points — awards for year 2010 not yet known — have been scaled up by 10/9). Despite its S-shaped form the black line is only *approximately* logistic.

Of particular interest are the values of the coupling constants. They are both positive indicating a win-win nature for the competition. In a symbiotic relationship each competitor benefits from the existence of the other, which is in line with the dynamics of scholarly research (each publication triggers more publications). But Americans benefit more when non-Americans win Nobel Prizes than vice versa. The ratio c_{xy} / c_{yx} is about 1.5 implying that one Nobel Prize won by a non-American will trigger 1.5 times more Nobel Prizes for

Americans than the other way around. This is counteracted to some extent by the smaller *a* constant for Americans.

The *a* constant reflects the species' ability to multiply. For products it specifies the product's attractiveness defined as

Attractiveness = e^a

In nature attractiveness represents the average litter size for a species.[10] If it is greater than 1, the species population grows; if it is less than 1, it declines. The values of *a* in Table III translate to attractiveness for Americans and all others of 1.5 and 1.7 respectively. This indicates that each American Nobel laureate will "brood" 1.5 new American Nobel winners whereas for all others this number is 1.7.

All in all, the number of American Nobel Laureates in the long run should stabilize around an average of 61.4 per decade barely higher than 60.6 for all others.

Table III - Results for Volterra-Lotka Fits

						Starting values	
a_x	b_x	c_{xy}	a_y	b_y	c_{yx}	Americans	Others
0.393138	0.01484	0.574901	0.533724	0.014418	0.384974	2.3717	59.8681

X^2 = 10.50
Degrees of freedom = 14
Confidence level = 72%

Approximate confidence level is calculated from the value of the reducedX^2. Only statistical errors considered.

5. Discussion

Competition arises when there are different entities vying for a limited resource. The two approaches considered in this paper, logistic growth and Volterra-Lotka, correspond to different competitive struggles. In the first one the limited resource is the total number of Nobel laureates that the US will ever claim. The implication is that this number is capped. In other words, there will be a time when all Nobel Prizes will be awarded to nationals from other countries. Up to that time, Americans will be elbowing each other to win prizes and the fewer left in their "niche" the harder it will be to win one.

Logistic growth descriptions have been successful when used with products filling their market niche, epidemics filling their niche of victims, and in general each time a niche is filled or emptied in competitive circumstances. The approach renders itself for fitting an S-curve on cumulated historical data.

In the second approach — Volterra-Lotka equations — the competition with another species is also taken into account. The niche now is all Nobel Prizes awarded annually, not only the ones destined for Americans. This competitive struggle can take many forms the most publicized of which is the predator-prey struggle in which the predator grows on expense of the prey but also depends on the prey so that when the latter diminishes in numbers the predator also diminishes and oscillations ensue. But with Nobel Prizes no oscillatory behavior is observed. The competitive struggle turns out to be a win-win relationship and following some substitution in the early 20th century the two species grow in parallel to a peaceful and stable coexistence in a symbiotic relationship.

Interestingly the US trajectory is S-shaped, which suggests that a logistic fit could have been a reasonable approximation but not on the cumulative numbers. The fit should have been on the numbers per unit of time. The limiting resource in this case would have been the annual number of American laureates. This number was zero at the turn of the 20th century and progressively grew to 8 by 2009 (6.4 on the average during the last 9 years). The meaning of competition in this picture would be that Americans elbow each other every year for one of their "quota" prizes that grew along an S-curve and in the 21st century reached a ceiling of 6.1.

6. Conclusions

Logistic S-curves are special cases of solutions to the Volterra-Lotka system of equations. The Volterra-Lotka Equations (3) reduce to the logistic Equation (2) whenever the coupling constants c_{ij} become zero. Whereas logistic growth describes competition only among the members of one species, the Volterra-Lotka system of equations handles competition also with other species. It is advisable to consult the Volterra-Lotka approach — whenever possible — even if one is interested only in logistic growth because it can shed light on how to apply the logistic-growth equation. In the US Nobel-laureates study the Volterra-Lotka solution dictates that a logistic S-curve should be fitted on the annual numbers and not on cumulative numbers. Had we done so we would have obtained an answer very close to the black S-shaped curve of Figure 3.

Deciding whether to fit S-curves on cumulative or on per-unit-of-time data is a crucial first step for all logistic-growth applications and constitutes treacherous terrain for inexperienced S-curve enthusiasts. I myself mastered it only later in my career.[11]
The forecasts for American Nobel laureates from the Volterra-Lotka approach are stable around an annual average of 6.1, comparable to the number of Nobel laureates won by all other nationalities together. Moreover the fitted parameters give rise to some interesting insights. The competition between Americans and all others for Nobel Prizes is of the win-win type. Locked in a symbiotic relationship both sides are winning but Americans are profiting more by 50%. At the same time, the ability of Nobel laureates to "multiply", i.e. the extent to which a Nobel laureate incubates more laureates, is lower for Americans than it is for other nationalities. One may ponder whether the roots of this last observation have something to do with the fact that chauvinistic traits tend to be more endemic in cultures with longer traditions.

All conclusions need to be interpreted within the uncertainties involved. From Tables I – II we see that the quality of the logistic fits worsens as the time window increases. Normalizing to the population improves the quality of the fits. In Table III a confidence level of 72% indicates that there is 7 out of 10 chances that the Volterra-Lotka description is the right way to analyze this competition, not very different from the last fit in Table II. For the intermediate future — ten to twenty years — the logistic normalized to reasonable population projections would result in forecasts compatible with those of the Volterra-Lotka approach. Still, I would choose Volterra-Lotka because it addresses a more general type of competition. In any case, long-term forecasts cannot be reliable and the whole exercise must be repeated with updated data sets in a couple of decades, by which time it may be appropriate to consider more than just two players.

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