# Genetic Re-Engineering of Corporations<sup>1</sup>

#### THEODORE MODIS

#### ABSTRACT

The work presented here constitutes a chapter in a forthcoming book by the same author entitled *What to Do When You Don't Know What to Do.* The approach uses the Voltera-Lotka equations to describe the competitive dynamics in a market niche occupied by two competitors. All types of competition are considered. Examples from industry demonstrate the possibility to alter the competitive roles by acting on the parameters of the equations. A methodology is given on how to guide and optimize advertising and image-building strategies.

#### Introduction

Industrial applications of biological models have a long history. One of the pioneers of this work was Alfred Lotka with his classic book *Elements of Physical Biology*. At the heart of all biological models lies the logistic growth, as dictated by Verhulst's equation:

$$\frac{dX}{dt} = a_x X - b_x X^2 \qquad \text{where } a_x, b_x, \text{ and } M \text{ constants}$$
 (1)

The solution of this equation is the ubiquitous S-shaped curve that enters extensively into every-day life. But Verhulst's equation deserves more credit than that. In its discrete form it is responsible of the whole science of chaos. Furthermore, as generalized in the Voltera-Lotka formulation, this equation can account for all types of interference between competing species. Finally, one might justifiably expect that in its full generality, the growth equation, in a discrete form, with cross terms to account for all inter-relations between competing species, would give a *complete* picture, in which growth, chaos, self-organization, complex adaptive systems, and many other trendy academic pastime ensue as special cases. But let us keep a lower profile here and concentrate in the simple and practical case of the two-competitor niche, something that has already been addressed in this journal.<sup>3</sup>

## The Shape of the Logistic

The S-shaped curve describes the growth in competition of a species population. The origin of competition is due to the fact that members of the same species elbow each other in a crowded niche. In the presence of more than one species, the S-curve law does not generally apply, because one species may interfere with the growth rate of another. More terms must be added to the mathematical formulation to take this interaction into account, and the S-shape pattern becomes distorted. Exception make one-to-one substitutions. They involve two competitors only and yet their "market" shares follow S-shaped patterns, see the case of cars and horses discussed in reference 2.

<sup>&</sup>lt;sup>1</sup> This article received the **Outstanding-Paper-of-the-Year Award** in the journal *Technological Forecasting & Social Change* where it was published.

There are two bends in the graceful shape of the celebrated S-curve. The first one (exponential rise) is due to the capability of the species to multiply. The second one (niche-saturation slow-down) is due to the competitive squeeze caused by the limited space.

**The first bend.** If you put a pair of rabbits on a fenced-off range, you can watch their population increase by going through the successive stages of 2, 4, 8, 16, 32,..., 2<sup>n</sup> in an exponential growth. If the average rabbit litter is greater than 2, you will see a steeper exponential growth. The same is true with products because they too have a capability to multiply. Depending upon its *attractiveness* every product sold will bring new customers. *Attractiveness* is the equivalent of the average rabbit litter. It is defined as:

$$Attractiveness = e^{a}$$

where a is the constant in Equation (1). The more products sold and the more attractive they are, the higher the rate of sales will be. Sales will grow at a constant percent rate—that is, exponentially—for a while, with a time constant which depends on the attractiveness. (If a product's attractiveness is smaller than unity, we are dealing with an unsuccessful product and its sales will quickly dwindle down to zero.) The first bend of the S-curve comes from the first term in equation (1).

The second bend. The rabbit-population explosion ceases when a sizable part of the niche becomes occupied; the same is true with products. If the growth equation is to be valid for late as well as for early times it must contain a term that represents the fact that the niche capacity is finite. This is the negative term in equation (1). The coefficient  $b_x$  expresses the strength of internal competition between members of the same species. In other words, it says that the percent rate of growth is also proportional to the still-empty space in the market niche. The second bend of the S-curve comes from the second term in equation (1).

# More than One Species in The Same Niche

Two parameters, the attractiveness and the niche capacity (or  $a_x$  and  $b_x$ ) fully determine the S-shaped pattern evidenced in the evolution of a species population diffusing in its ecological niche. But what happens if besides rabbits we also have sheep on the range? After all, sheep also eat grass and in greater amounts than rabbits. Their presence will certainly suppress the rabbit population explosion. Worse yet, what happens if there are foxes? Competition between rabbits and sheep is not the same as between rabbits and foxes. Just think of the fact that faced with a finite amount of grass, sheep would probably lament at the rapid multiplication of rabbits, while foxes would undoubtedly rejoice.

It all has to do with how one competitor influences the growth rate of the other. Sheep and rabbits have a negative effect on each other's population by reducing each other's food supply, but while foxes damage rabbit populations, the latter have a positive influence on fox populations. Whenever there is more than one competitor in the same niche, we must consider the interaction between them, namely, how one's rate of growth depends on the existence of the other. We then need to introduce a third term in the growth equation to take this coupling into account. For two competitors *X* and *Y* the equations become:

$$\frac{dX}{dt} = a_x X - b_x X^2 + c_{xy} XY$$

$$\frac{dY}{dt} = a_y Y - b_y Y^2 + c_{yx} YX$$
(2)

The values of  $c_{xy}$  and  $c_{yx}$  are related to the overlap, how much one steps on the feet of the other, or in other words, how many sales you will lose (or win), because your competitor won one. With the system of equations (2) we can formulate a measurement for one's ability to attack, counterattack, or retreat, as the case may be.

# Attacker's Advantage, Defender's Counterattack

The attack of a new species against the defenses of an incumbent lies at the heart of corporate marketing strategies. This kind of struggle had already been rigorously formulated by biologists and ecologists. Farrel tells of how in the 1930s George Gause, at Moscow's Zoological Museum, studied the competition between a traditional brewer's yeast and one used in Ukraine to make the refreshing milk drink called kefir, popular in Asian and Middle-Eastern countries. He first grew the two yeasts in isolation and observed the S-shaped natural growth pattern for each. He then put them together in the same test tube and let them compete for the same food. He found that each influenced the other's growth. But the brewer's yeast is tolerant to the alcohol that is produced as it grows; the kefir yeast is less so. In a mixture, this gave the brewer's yeast an increasing advantage as fermentation proceeded, and it outgrew its competitor. Simple S-shaped curves did not describe the growth processes well, but the Volterra-Lotka mathematical formulation involving coupling constants did.

The discrete forms of Equations (2) have been worked out by Leslie as follows:<sup>5</sup>

$$X(t+1) = \frac{l_{x} X(t)}{1 + b_{x} X(t) - A b_{x} Y(t)}$$

$$Y(t+1) = \frac{l_{y} Y(t)}{1 + b_{y} Y(t) - D b_{y} X(t)}$$
(3)

where 
$$l = e^a$$
 and  $b = \frac{b(e^a - 1)}{a}$  for  $X$  and  $Y$ 

and

$$A = \frac{c_{xy}}{b_x} \qquad D = \frac{c_{yx}}{b_y}$$

Assuming that X is the incumbent and Y the attacker, we can define A as the attacker's advantage, and D as the defender's counterattack.

A quantifies the extent to which the attacker inhibits the ability of the defender to keep market share. D quantifies the extent to which the defender can prevent the attacker from stealing market share. The business strategy and tactics of attack and counterattack have been qualitatively described by Peter Drucker<sup>7</sup> and especially by Richard Foster<sup>8</sup>—director at McKinsey & Co. The nature of the attacker's advantage has been clearly established by Cooper and Kleinschmidt<sup>9</sup>—professors respectively in industrial marketing and technology management, and in marketing and international business—who studied over 200 new products and determined that the most significant parameter in gaining market share is a "superior product that delivered unique benefits to the user." This and price considerations dictate the magnitude of the A.

Under attack, the defender redoubles its own efforts to maintain or improve its position. A high value for the *D* implies a face-on counterattack within the context "we do better what they do". An effective counterattack, however, with long-lasting survival-sustaining consequences implies eventual adoption of the new technology, some sort of death for the old company, which is painful to assimilate culturally. Companies hesitate to embark on such undertakings. Because of this hesitation, Foster refers to defender's counterattack as the defender's *dilemma* and cites tens of examples in which a defender refused to acknowledge, or reacted too late, to an attacker's onslaught. A classical case was NCR's belated and traumatic transition to computerized cash registers.

Figure 1 shows a more recent example, the competition in the Greek mobile-telephone market, a two-competitor struggle. The two firms launched their products simultaneously. One firm, Telestet, became an early market leader thus assuming the role of the defender. But the coupling

# MOBILE-TELEPHONE SALES IN GREECE

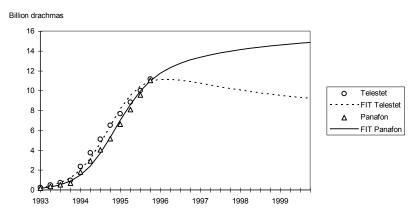


FIGURE 1 We see quarterly sales for the two competitors of the Greek mobile-telephone market. Despite early dominance by Telestet, the model successfully predicted the shift in Panafon's favor by mid 1996.

parameters, determined from the data, were both negative, A = -0.8 and the D = -0.6. They indicated much overlap and fierce competition of the sheep-rabbit nature. Every time Panafon would close a sale, Telestet would loose 0.8 potential sales, and every time Telestet would close a sale, Panafon would loose 0.6 sales. The difference was crucial. The model showed curves that eventually deviated from S-shapes. With data up to the end of 1995, the model's prediction was that Panafon would become leader within a few months. By mid 1996 Panafon's market share had been established higher than Telestet's.

Kristina Smitalova and Stefan Sujan, professors of mathematics at Comenius University and the Slovac Academy of Science respectively, in Bratislava, Slovakia, studied and classified the various coupling schemes in a rigorous way. <sup>10</sup> They distinguished and labeled six ways in which two competitors can influence each other's growth rate, according to the sign of the two coupling parameters involved. We tabulate them in the table on next page.

- -- PURE COMPETITION occurs when both species suffer from each other's existence.
- + PREDATOR-PREY occurs when one of them serves as direct food to the other.
- ++ MUTUALISM occurs in case of symbiosis or a win-win situation.
- + 0 COMMENSALISM occurs in a parasitic type of relationship in which one benefits from the existence of the other, who nevertheless remains unaffected.
- 0 AMENSALISM occurs when one suffers from the existence of the other, who is impervious to what is happening.
- 0.0 NEUTRALISM occurs if there is no interaction whatsoever.

Pure competition is what we have between rabbits and sheep. Each one influences negatively the growth of the other, but not with the same importance (sheep are fewer but eat more). Market examples are the mobile-telephone case mentioned above, or the competition between different-size computer models.

Predator-prey is the case of cinema and television. The more movies made for cinema, the more television will benefit, but the more television grows in importance, the more cinema suffers. Telefilms are not shown in movie theaters. Had been no legal protection, (restricting permission to broadcast new movies), television would have probably "eaten up" the cinema audience.

A typical case of mutualism is software and hardware. Sales of each one trigger more sales for the other.

Add-ons and accessories such as car extras constitute an example of commensalism. The more cars sold, the more car accessories will be sold. The inverse is not true, however; sales of accessories do not trigger car sales.

Amensalism can be found in the case of ballpoint pens and fountain pens described in detail below. The onslaught of ballpoint sales seriously damaged fountain-pen sales, and yet the ballpoint-pen population grew as if there were no competition.

Examples of neutralism are encountered in all situations in which there is no market overlap. For example, a sports store that sells both swimming-wear and ski-wear. Depending on the geography there might be a negative correlation of seasonal origin, but the sales of one do not in general affect the sales of the other.

## **Competition Management**

The intriguing fascination of the marketplace is that *the nature* of competition can be changed over time. For some business people achieving a change in the competitive roles is perhaps more handsomely rewarding than making profits. It is something that species in nature cannot do. Rabbits will never eat meat, and whenever humans tamper in such areas, either academically (genetic engineering) or industrially (mad-cow disease), they are invariably criticized, justly or unjustly.

But things are different in industry. In contrast to the jungle, a technology, a company, or a product does not need to remain prey to another forever. The competitive roles can be radically altered with the right decisions at the right time. External light meters, used for accurate diaphragm and speed setting on photographic cameras, enjoyed a stable *commensal* relationship with cameras for decades. As camera sales grew, so did the sales of light meters. But there came a time when technological developments enabled cameras to incorporate light meters into their own box. Soon the whole light-meter industry became prey to the camera industry. Sales of external light meters diminished while sales of cameras enjoyed a boost, and the relationship passed from *commensalism* to a *predator-prey* one.

The struggle between fountain pens and ballpoint pens, mentioned earlier, had a happier ending. Another case of genetic re-engineering in the marketplace, the substitution of ballpoint pens for fountain pens as writing instruments went through three distinct stages.

Before the appearance of ballpoint pens, fountain-pen sales were growing undisturbed to fill the writing-instrument market. They were following an S-shaped "rabbit curve" when the ballpoint technology made its appearance in 1951. As ballpoint sales picked up, those of fountain pens declined for the period 1951-1973. Ballpoint pens did not belong to the same species, neither did they constitute a one-to-one substitution, and yet they cut deep into the fountain-pen sales. A simple S-shaped pattern could not have described this transition but the Volterra-Lotka equations did, with attacker's advantage A = -0.5 and defender's counterattack D = 0 (see Figure 2). These numbers imply a competitive advantage for ballpoint pens, which by winning one customer inflict losses of half a customer to fountain pens. Fountain pens staged a counterattack by radically dropping prices for many years. Their average price dropped as low as seventy-two cents. But the counterattack was ineffective—D remained equal to zero. While counterattacking fountain pens lost market share and embarked on a well-established extinction course.

Eventually the prices of fountain pens began rising. The average pen price in the US reached \$3.50 in 1980 and continued rising. In 1988 a Mont Blanc Masterpiece Diplomat retailed at \$280, while a Waterman Le Mans 100 Briarwood cost \$400. The fountain pen underwent what Darwin would have described as a "character displacement" to the luxury niche of the executive pen. The strategy of fountain pens since the early 1970s has been a retreat into non-competition. Indeed, A and D must both equal to zero for the Volterra-Lotka equations to do justice to the sales data of writing instruments in this period. In other words, we have two species that do not interact—neutralism—but each follows a simple S-shaped growth pattern. As a consequence fountain pens have secured for themselves a healthy and profitable market niche. Had they persisted in their competition with ballpoint pens, they would have perished.

#### THE STRUGGLE BETWEEN BALLPOINT AND FOUNTAIN PENS 1929 1934 1939 1944 1949 1954 1959 1964 1969 1974 1979 1984 1989 1994

FIGURE 2 Sales of fountain pens and ballpoint pens in the US. The lines are the biological-model descriptions. Before 1951 and after 1973 we see S-shaped patterns for each competitor. Between 1951 and 1974 we see a typical *amensal* type of competition where the attacker has an advantage (A = -0.5), while the defender's counterattack is null (D = 0).

Having quantified the competitive mechanisms during the period 1951-1973, it is amusing to play the following scenario. What would have happened had fountain pens undergone their character displacement five years earlier? The model's answer is a significantly higher number of sales for fountain pens today. Is it believable?

One could argue so. Fountain pens would have embarked on an upward trajectory earlier, starting from a stronger position. Enhanced fountain-pen content in everyday life could have cultural repercussions over time, and produce societal preferences and habits. At the end, a more favorable average-citizen disposition could have conceivably led to a more important role for the fountain pen today. Consequently on the average, their price would have to rise less, and their image a little more popular and a little less exclusive.

Character displacement, otherwise referred to as Darwinian Divergence, is a classical way to diminish the impact of competition.\*

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<sup>\*</sup> In his book *Born to Rebel*, Frank Sulloway—a history-of-science turned sociologist—at MIT's program in Science, Technology, and Society, proves that throughout history first-born children have become conservative while laterborns revolutionary. First-born children end up conservative because they do not want to lose any of the only-child privileges they enjoy. But this forces laterborns into becoming rebellious, to differentiate themselves thus minimize competition and optimize survival in the same family.

#### Finding The Magic Advertising Message

The Volterra-Lotka equations require three parameters per competitor to describe growth in a two-competitor niche. One parameter represents the ability to multiply, another the size of the niche, and the third one interference from the other competitor. Consequently, we have three choices for action; or six, if we want to take into consideration the parameters of the competitor. To increase the prospects for growth then we could try to change one or more of the following:

- The product attractiveness (increase ours or decrease theirs)
- The size of the market-niche (increase ours or decrease theirs)
- The nature of the interaction (increase our attack or decrease their attack)

#### THE SIX DIMENSIONS OF ADVERTISING ACTION

Attractiveness		Niche size	Competition	
WE:	Our products are good	7 You need our products	<b>¥</b> We are different	
THEY:	Their products are not good	You do not need their products	We do better what they do	

FIGURE 3 The six possible independent advertising messages according to the biological model.

Each direction of action in principle affects only one parameter.\* But it is not obvious which change will produce the greater effect; it depends on the particular situation. The concrete actions may include performance improvements, price changes, image transformation, and advertising campaigns. Performance and price concern one's own products only, but advertising via the appropriate message can in principle produce an effect on all six of the parameters. The question is how much of an effect will a certain effort produce. Some advertising messages have proven significantly more effective than others. Success is not necessarily due to whim, chance, or other after-the-fact explanations based on psychological arguments. The roles and the positions of the competitors at a given point in time determine which advertising message will be the effective one. We can illustrate the effectiveness of advertising messages with an example typical of competitive technological substitutions: woven carpets and tufted carpets.

Woven carpets were made on a loom in a manner similar to plain cloth, except that extra wrap yarns were introduced and raised by wires to form loops. Most of today's carpets are made with needles that punch loops through a backing and retreat to leave tufts. Examining the backing of a typical modern carpet reveals the use of glue to hold the tufts in place. This revolution in carpet making began in the 1950s. Tufting changed the requirements for the yarn. Long, continuous filaments were preferred, as they didn't pill or fuzz. Wool yarns have fibers as short as the annual growth of a sheep's hair. This put a fiber such as nylon in a very good position, especially when DuPont invented a bulked form of continuous fiber. The combination of this and tufting created a new "species" that satisfied a growing demand for carpeting and caused the displacement of woolen-woven carpets by nylon-tufted ones. 12

The model description of the data indicates that the A = -2.2 and the D = -2.6. This is a typical situation of *pure competition* between two similar-species contenders even though the attacker sells in greater numbers. The fate of the defender is eventual extinction.

<sup>\*</sup> The truly independent variables are: attractiveness, time constant, and occupancy, as defined below. They are related to but not one-to-one to the parameters accessible to change. Consequently, some parameters will change together.

A. The *time constant* of the multiplication process is defined as: time constant = 1 / log(attractiveness)

B. The occupancy is defined as:  $occupancy = 1 / [(time\ constant)\ x\ (niche\ size)]$ 

Could the makers of woolen-woven carpets have secured for themselves a market niche as did fountain pens? And if it were possible, what line of action should have they followed? Let us explore alternative lines of action—via advertising campaigns—and their effectiveness in shaping a different future for woolen carpets back in 1979. Let us rate the different scenarios stemming from changing the six parameters one at a time *by the same amount*, which to a first approximation can be taken as equivalent to comparable effort investment. It is a sensitivity study on the effectiveness of the corresponding advertising message.

Figure 4 shows two of the six possible results. Effective campaigns would be those which emphasized attractiveness and differentiation with messages like "wool is good" and "wool is different from nylon." On the other hand, a counterattack along the lines: "wool is better than nylon", would have been very ineffective.

#### SUBSTITUTING NYLON-TUFTED CARPETS FOR WOOLEN-WOVEN ONES.

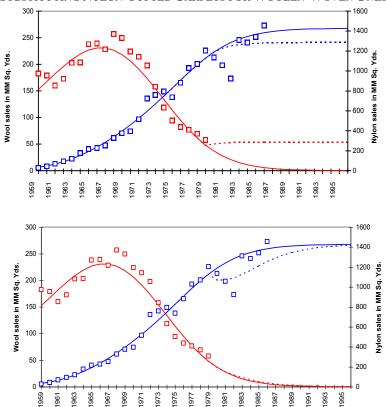


FIGURE 4 The dotted lines indicate two scenarios of comparable change in the respective parameters. On the top following an advertising campaign under the slogan "wool is different from nylon"; on the bottom under the slogan "wool is better than nylon". 13

The table below shows the complete list of possible advertising messages and their effect on the evolution of wool and nylon sales in 1979. Each message represents an *independent* direction in which the full traditional advertising machinery would have to be launched. There should be no cross-talk between directions. For example, to obtain maximum benefit from the "wool is good" campaign one *should not* mix connotations such as "wool is better than nylon" or "nylon is bad". Each message would have to be developed and exploited separately.

Although the detailed execution of the advertising campaign (media, wording, style, etc.) remains crucial, the effectiveness ratings of the above directions come in a non-obvious if not surprising order, and could not have been arrived at by intuitive or other methods used by advertising agencies. Furthermore, the order may be completely different at another time or another market.

Here are the results of the complete sensitivity analysis following six scenarios played for the wool-nylon case study. Playing the scenarios from wool's point of view, we measure effectiveness according to how much wool benefits.

## Sensitivity Analysis

Effectiveness	Wool	Nylon  little compromised
highest	slowly rising from '79 level	
high	stabilizes at '79 level	little com- promised
medium	stabilizes at 0.5 of '79 level	huge loss of market
poor	stabilizes at 0.3 of '79 level	serious loss of market
negligible	null	temporary losses only
null	no effect	no effect
	highest high medium poor negligible	high stabilizes at '79 level  medium stabilizes at 0.5 of '79 level  poor stabilizes at 0.3 of '79 level  negligible null

Finally there is a way to assess the size of the advertising investment called for. An advertising campaign along the lines of "our product is good" impacts the product's attractiveness just as price dropping does. (Price can be quantitatively related to attractiveness via the price elasticity). The costs that would have incurred from price dropping alone can thus be compared to those of an advertising campaign that would achieve the same result. Naturally, this assessment may result in an overestimate or an underestimate depending on how the advertising campaign in question rated to the "our-product-is-good" alternative in the sensitivity analysis. We would like to point out, however, that if we relied on price dropping alone for the survival of woolen carpets, their price would have to be dropped by more than 100 percent!

The case of the Greek mobile-telephone market mentioned earlier—Figure 1—is more malleable, however. As we indicated, it was possible for Telestet to have anticipated its eventual loss of the leading position. Had its managers taken action in the beginning of 1996 toward increasing the attractiveness of their products by 10 percent—for example, by dropping prices by 8 percent—they would have safeguarded their lead. Of course, Panafon may have rapidly responded in kind, but this is what the business game is all about, and to a large extent, it can be successfully, and painlessly, simulated on your personal computer!

# Who Is Afraid of The Big Bad Wolf?

The S-curve model enhanced with two-species interactions, as presented above, accounts for the three most fundamental factors that shape growth: the attractiveness of one's offering, the size of its market niche, and the interaction with the competitor, (in cases where there is more than one competitor, one can always reduce the situation to a two-player picture by considering the major competitor only, or by grouping all others together). Naturally, there are other factors that influence growth, such as channels, distribution, market fragmentation, total market growth, market share, frequency of innovations, productivity in the ranks, and organizational and human-resource issues. Many of them can be expressed as combinations of the three fundamental ones. Alternatively, one could envisage elaborations of the model—adding more parameters—to take more phenomena into account.

As it stands, the model provides the base line, the trend on top of which other higher-order effects will be superimposed. It guides you through effective genetic manipulations of the competitive roles in the marketplace. It should be used as a front end to what is usually done. The model works equally well for products, for corporations, for technologies, or for whole industries. Only the time scales differ. The pleasure is all the strategist's, who now has a quantitative, science-based way to understand the crux of the competitive dynamics and to anticipate the consequences of possible actions.

Just think of it,—at this very moment there may be a cost-effective way to terminate the state of being prey to the voracious competitor who has been feeding persistently on your achievements.

The author is indebted to professor Carl Pistorius of the University of Pretoria for fruitful discussions on this topic.

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<sup>&</sup>lt;sup>2</sup> One can find numerous examples in Modis, T., *Predictions*, Simon & Schuster, New York, 1925.

<sup>&</sup>lt;sup>3</sup> Farrell, C. 1993. A Theory of Technological Progress. *Technological Forecasting & Social Change* vol 44, no 2, 161.

<sup>&</sup>lt;sup>4</sup> Farrell, C., Survival of The Fittest Technologies, New Scientist 137, 6 February 1993.

<sup>&</sup>lt;sup>5</sup> Leslie, P.H., A Stohastic Model for Studying The Properties of Certain Biological Systems by Numerical Methods, Biometrika 45, 1957

<sup>&</sup>lt;sup>6</sup> This is how Christopher Farrell defines the attacker's advantage, and the defender's counterattack in reference 3, above.

<sup>&</sup>lt;sup>7</sup> Drucker, P.F., The Discipline of Innovation, *Harvard Business Review*, May-June, 67-72 (1985).

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<sup>&</sup>lt;sup>11</sup> This case study has been discussed by Farrell, see reference 3 above. The data for this graph have been digitized from a graph therein. Missing-year data have been interpolated.

<sup>&</sup>lt;sup>12</sup> The case study originates with Farrell, see reference 3 above.

<sup>&</sup>lt;sup>13</sup> The data for this graph have been digitized from a graph in Farrell, see refernce 3 above.